

5th BSME International Conference on Thermal Engineering

Towards a transient simulation of thermo-acoustic engines using an electrical analogy

Paul H. Riley^a
The University of Nottingham, Department of Electrical and Electronic Engineering, Nottingham, NG7 2RD UK.

Abstract

An electrical analogue of a looped-tube, thermoacoustic engine is presented and simulated using a graphical input, off-the-shelf network analysis tool called TinaTM. The analyses are able to show both transient and non-linear behaviour. The recently documented phenomenon from the Score-Stove project of amplitude modulation of the pressure wave, called squegging has been simulated. By use of elements such as Zener diodes to simulate turbulent behaviour, non-linear effects can be seen. Thus the simulation shows the effect of many frequencies, not just the fundamental as is the case with conventional thermoacoustic simulations. Gradual cold start-up, and hot start conditions and a simple technique to ensure mathematical convergence on a non-zero solution are presented.

© 2013 The Authors. Published by Elsevier Ltd.

Selection and peer review under responsibility of the Bangladesh Society of Mechanical Engineers

Keywords: Non-linear; thermoacoustics; transient-behaviour; Score-Stove; Tina

Nomenclature

ρ_{tag}	Density of the thermo-acoustic gas at STP (Kg/m ³)	Tc	Regenerator temperature ambient side (°K)
C	Speed of sound (m/s)	tc	Regenerator temperature ambient side (°C)
C'	Mechanical Volume's Equivalent electrical capacitance ^b (F)	Th	Temperature of the hot side of the regenerator (°K)
Dp	Pipe Diameter (mm)	th	Temperature of the hot side of the regenerator (°C)
L'	Acoustic Inertance's Equivalent electrical (H)	v	Particle velocity (m/s)
Lp	Pipe length (m)	Z0	Characteristic duct impedance, rho x density of the thermo-acoustic gas (Ns/m)
Pa	Atmospheric pressure (Bar)	Z0'	Electrical characteristic impedance (Ohms)
Pm	Mean pressure in TAE (Bar)	τ	Tau, Regenerator thermal time constant (seconds)
R'	Equivalent electrical resistance ^c (Ohms)	ω	Frequency (Rads per second)
Rp	Pipe loss (Ns/m)	$\omega\tau$	Regenerator non-dimensional figure of merit # [1]
Rr	Regenerator loss (Ns/m)		
Sc	linear alternator cone area (m ²)		
Sd	thermo-acoustic duct (pipe) area (m ²)		

^a Tel: +44 (0) 115 9515356, email: paul.riley@nottingham.ac.uk

^b This is the analogue of a mechanical volume. Note volumes are always connected to ground in the electrical circuit. Series capacitance has no acoustic equivalent.

^c Equivalent to mechanical loss

1. Introduction

There are a number of packages currently available for simulating thermo-acoustic engines (TAE), the most well known being DeltaEC [2] written by Swift of Los Alamos laboratories in the US. Most give a reasonably accurate estimation of performance by simulating steady state one dimensional flow. There has also been some success using CFD code [3], [4], analyses using high order differential equations [5] and experimental infrared imaging [6] in predicting non-steady state performance, but setting up the models can be time consuming and difficult to validate. There is much work on electrical analogues in the literature [7] and code already exists to solve the complex mathematics. In real world applications, such as the Score-Stove™ [8], [9] when the heat is produced by a wood flame, understanding of transient behavior would give a result that more closely predicts system performance. Additionally, a transient simulation would enable a better understanding of a phenomenon called squegging^d [10] [11] [12]. Yu, Jaworski et al [13] have experimentally observed this TAE behavior, but it has not been simulated to our knowledge. Many TAE's generate harmonics that conventional simulations do not model. The electrical analogue has shown some success in predicting this effect as well. This paper presents an electrical analogue of a TAE that uses a commercially available electrical network solver called Tina™ to give transient and steady state behavioral response. The simulation presented can predict qualitatively typical transient behavior. Other network solvers can also be used but Tina™ provides an easy to use graphical interface that makes circuit changes both quick to make and more easy to relate to the physical geometry.

2. Circuit elements

The TAE is modelled using a variety of analogues, described in the following sections. Circuit components have been labelled appropriately to their analogue.

2.1. Mathematical convergence



Tina™ (version 9.3 and above) will converge with most circuits, and if not, small changes in values can secure convergence. The Resistor models also include a noise element; hence the TAE simulations will sometimes start automatically. However, to ensure operation a starting pulse of 0.1mA for 1ms should be used as shown on Figure 1. This has little effect on accuracy but ensures correct start-up each time.

Fig. 1. Start pulse

2.2. Acoustic

Acoustic circuits can be simulated with electrical components in a variety of ways, each with advantages and disadvantages. This electrical analogy of a thermo-acoustic engine uses the following transforms as the author believes that in the Score [9] application they are the most understandable and are shown on Table 1. Table 2 outlines the thermal analogs.

Table 1

Acoustic	Electrical	Thermal	Electrical
Pressure x Sd	Voltage	Temperature	Voltage
Particle Velocity (v)	Current	Heat flow	Current
Acoustic power	V x I	Thermal Mass	Capacitance
Mass_LA x Sc/Sd	L'	Thermal resistance, Rθ	Resistance
rho * length of pipe /Sd	L'/metre (transmission line inductance)		
volume (in m^3) /rho/c^2/Sd^2	C'		
Pressure * Sd/Particle velocity	R'		

Table 2

^d Squegging is a term normally used in Electronic RF oscillators to describe unwanted amplitude modulation (AM) of the carrier wave. Similar AM has been seen in TAE's so the term is appropriate to use in TAE applications.

2.3. Sources

There are a number of ways to generate current and voltage. These are called controlled sources. In a TAE, they are used to simulate various parts of a TAE. Fig. 2 is a current controlled current source; a velocity amplifier, Fig. 3 a voltage controlled current source; pressure to velocity converter, Fig. 4 and Fig. 5 are more complex controlled sources with the transfer function determined by an equation.

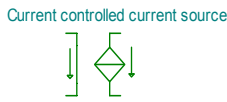
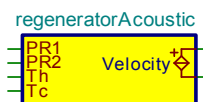


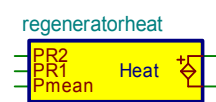
Fig. 2. CCCS



Fig. 3. VCCS



$$\text{Velocity} = \frac{(V(\text{PR1}) - V(\text{PR2})) * ((V(\text{Th}) + 273) / (V(\text{Tc}) + 273) - 1)}{1}$$

Fig. 4.
Four input controlled current source

$$\text{Heat output} = \frac{(V(\text{PR2}) - V(\text{PR1})) * (V(\text{PR1}) - V(\text{Pmean})) * 20}{1}$$

Fig. 5.
three input controlled current source

2.4. Simple regenerator

The regenerator is effectively a flow modulating device with gain being a function of $T_h - T_c$. In his book [14], Swift models this as a current controlled current source as thermo-acoustically the regenerator is a velocity amplifier. In Tina this is modelled as a voltage controlled current source as this has proven more mathematically stable. The voltage input is scaled by the regenerator loss, to give the equivalent current controlled current source. The circuit is shown on Fig. 6. Regenerator volume is modelled by a capacitance. “Regen loss” is the electrical equivalent of the regenerator flow resistance, and regenerator volume in the total volume of the regenerator itself plus any additional volume associated with the regenerator housing.

The simple model assumes infinite heat flow, so any circuit that uses this method has to limit current flow (particle velocity) by another means. Fig. 7 shows Back to back Zener diodes that limit pressure to perform this function. Note the devices chosen should have a very low internal resistance to prevent the current source from forcing the voltage (pressure) to an end stop at infinity. Parallel diodes can also perform this function, but the pressure is limited to 0.7 volts equivalent.

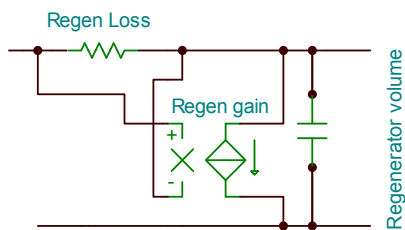


Fig. 6. Simple regenerator acoustic model

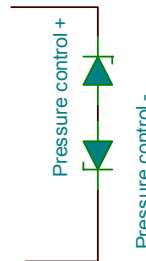
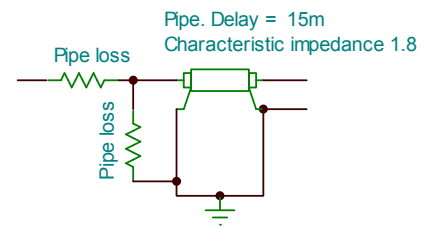
Fig. 7. Non-linear pressure
control

Fig. 8. Feedback pipe with losses

2.5. Feed back pipes

Pipes are modelled using a lossless transmission line, of characteristic impedance Z_0' combined with series and parallel resistors to simulate pipe loss. This method was found to converge more quickly than using the lossy transmission line component, and it is easier to understand for the non-electrical engineer.

The pipe length is simulated using a delay, where $Delay = \frac{L_p}{c}$ and

$$Z_0 = \text{Characteristic impedance} = c S_d \frac{P_m}{P_a} \rho_{tag}$$

2.6. Simple linear alternator

Wakeland [15] describes a typical Linear Alternator analogue that can be used in the TAE. Fig. 9 describes a simplified model for a loudspeaker working as a linear alternator where the speaker resonant frequency is less than the TAE operating frequency. When operating above resonance, a speaker behaves as an inductance. The resistance represents mechanical and electrical losses as well as load resistance. Power in the resistor therefore represents total acoustic power absorbed.

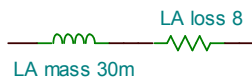


Fig. 9. Simple linear alternator model, working above resonance



Fig. 10. Reference conditions are represented by the ground symbol

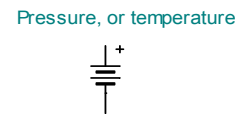


Fig. 11. Conditions away from reference use a voltage source.

2.7. Measurement

Tina™ allows the operator to place voltmeters (pressure transducers), ammeters (particle velocity) and power meters around the circuit. The transient solver can show start up, steady state operation or cool down characteristics depending on the time chosen for the start and finish of the simulation. Fig. 12 shows a voltmeter that reads pressure x duct area. (the equivalent of force). By the addition of an ideal transformer with turns ratio = S_d , Fig. 13 shows how to obtain a direct reading of pressure in real units, for example Pascals.

In Fig. 14 particle velocity can be read directly. Power is computed (Fig. 15) by inserting the horizontal wire into a velocity circuit, and the vertical wire to measure pressure. Reference conditions, for example 0°C, 0 Bar gauge etc. are defined using the ground symbol, Figure 10. Conditions away from reference are modelled using a current source (for velocity and heat flow) or voltage source as shown on Fig. 11.



Fig. 12. Voltmeter reads pressure x S_d

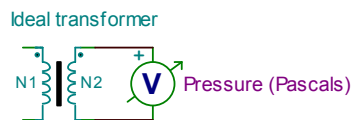


Fig. 13. Ideal transformer converts to real units.

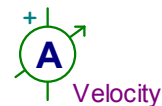


Fig. 14. An ammeter displays particle velocity in m/s

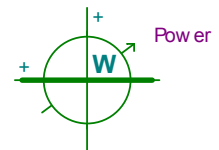


Fig. 15. Power measurement.

3. System

The models described below have all been tested using Tina version 9.3 obtainable from here: <http://www.designsoftware.com/home/English/&> . (Note earlier versions of Tina may not converge mathematically)

3.1. Simple acoustics-only analogue

Fig. 16 is a diagram of a single regenerator, looped tube TAE and Fig. 17 represents the analogue that will self oscillate. Heat flow is excluded from this model. The tuning volume, if short compared with the wavelength can be modelled with a

volume (capacitor), other elements are as described above. A simple resistor models the thermal buffer tube (TBT) and the secondary heat exchanger is not included.

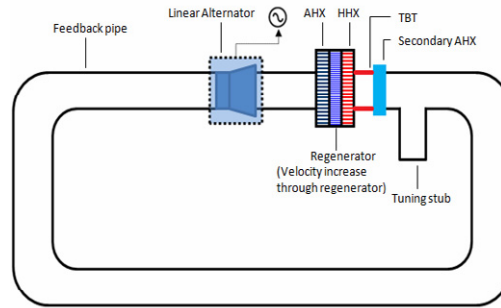


Fig. 16. Diagram of single regenerator TAE

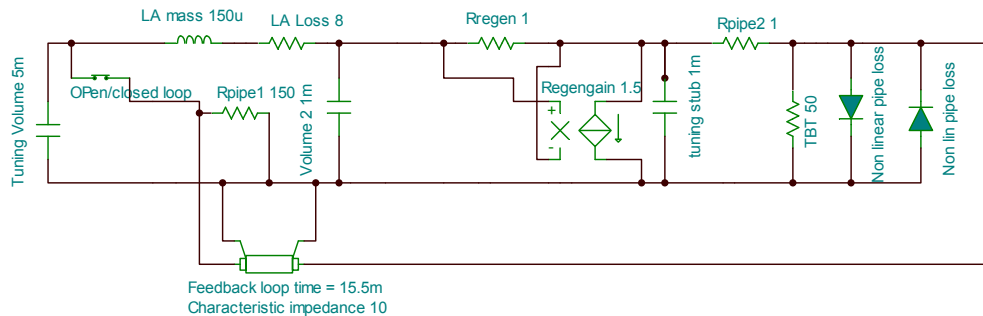


Fig. 17. Single regenerator TAE analogue

The open/closed switch is used to perform swept frequency measurements to understand open loop system performance to assist in taking the parameters of a non functioning TAE into the oscillating region. This function is outside the scope of this paper and is kept closed in the results described below.

3.2. Single regenerator including thermal

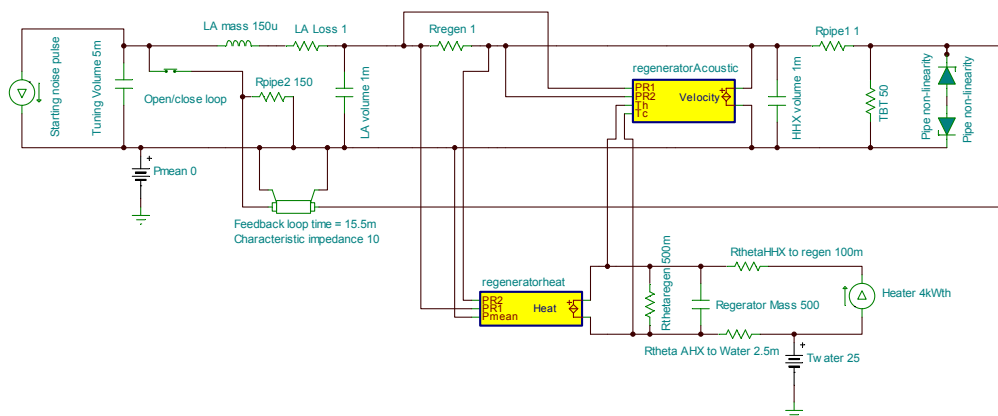


Fig. 18. Single regenerator TAE including thermal modelling

There are two analogues shown on Fig. 18. In the upper section the VCCS of Fig. 17 is replaced by the more complex velocity generator that models the acoustic velocity amplification and temperature difference characteristics of a regenerator. The lower section models the thermal characteristic of the regenerator. Ambient temperature is set to 25°C by the voltage source and heat exchanger performance by the thermal resistances and thermal mass (capacitor). A 4 kW constant power heat source is assumed. When oscillation starts, the velocity through regen 1 is multiplied by the pressure to obtain regenerator acoustic power, this multiplied by a constant (effectively the % of Carnot and is empirically derived in this model) then extracts heat from the thermal circuit which, acting in a feedback loop, limits the regenerator temperature.

By adding instrumentation to Fig. 18, the important system parameters can be measured. Fig. 19 shows the complete circuit used in the simulation results below. These circuits can be found on-line at this reference [16].

Single regenerator with instrumentation

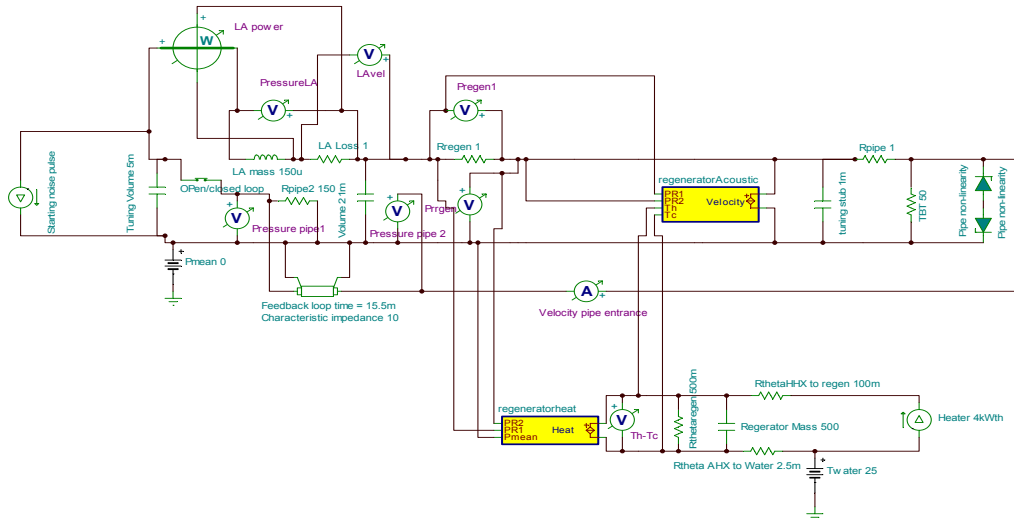


Fig. 19. Instrumented TAE analogue.

4. Simulation results

The circuit on Fig. 17 produces the start-up shown on Fig. 20. The circuit in Fig. 19 produces all the results in subsequent figures. With pipe non-linear loss (the zener diodes) set to a high value (=100), the circuit works in an essentially linear mode and squegging occurs (Fig. 23). Lowering pipe non-linear loss to a low value (=15) highlights the effect of large harmonic generation (Fig. 24). With normal operation (=50), Fig. 21 and Fig. 22 show the performance characteristics of typical of a looped tube TAE when started from a hot condition.

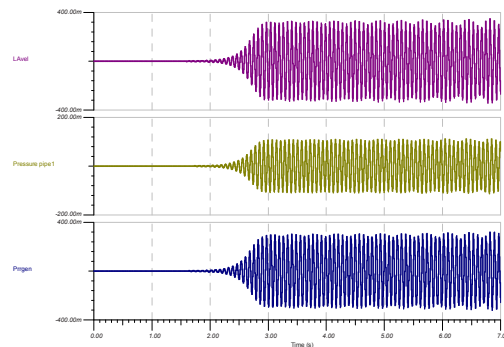


Fig. 20. Normal start-up, acoustic only

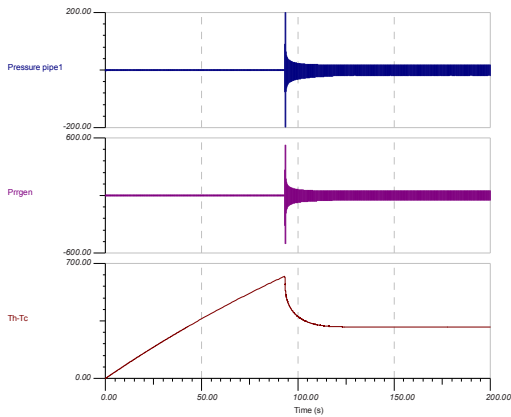


Fig. 21. Hot Start up response.

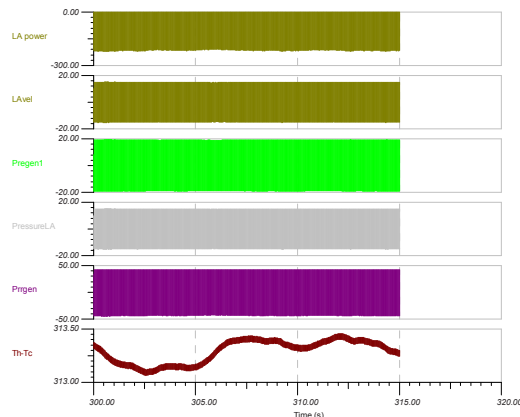


Fig. 22. Expanded Fig. 21 showing ~steady state operation

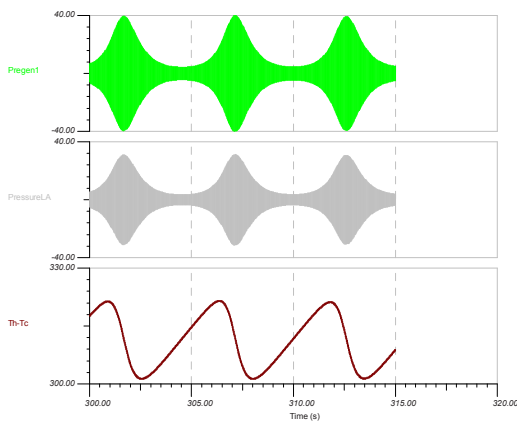


Fig. 23. Heavy squegging in what should be a steady state condition.

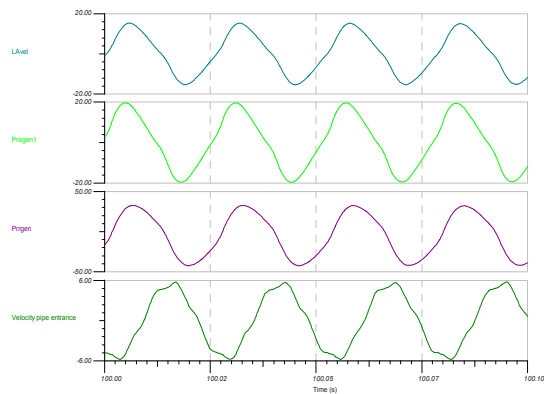


Fig. 24. Harmonic generation (non-sinusoidal waveforms)

5. Conclusions

Both transient and pseudo-steady state behaviour of a thermo-acoustic engine has been simulated using an electrical analogue. The circuit exhibits, in a qualitative way the unusual effects of squegging and harmonic generation seen in experimental thermo-acoustic engines but not previously simulated. The circuits are easy to change and the diagrams are easily understood with reference to physical geometry. Most of the circuits described have converged mathematically using Tina™ version 9.3 software. The start-up regime can be contrary, but use of a small staring impulse ensures correct operation.

Future work will look at the minimum physical features required to simulate a working TAE so that experimental results can be quantitatively compared with the simulations. The method should enable rapid simulation and analysis of TAE engines including more complex ones of the multi-regenerator variety without the large amount of training normally associated with current TAE simulators.

Acknowledgements

Thanks to EPSRC grant EP/J013986/1, my Score colleagues and special thanks to Scott Backhaus of Los Alamos and Kees deBlok of Aster Thermoakoestische Systemen.

References

- [1] “Low operating temperature integral thermo acoustic devices for solar cooling and waste heat recovery” *Acoustics* 08, K. De Blok.
- [2] <http://www.lanl.gov/thermoacoustics/DeltaEC.html>
- [3] “Simulation of a traveling-wave thermoacoustic engine using computational fluid dynamics”, J.A.L.A. Nijeholt, M.E.H. Tijani, S. Spoelstra. *J. Acoust. Soc. Am.*, 118 (4) (2005), pp. 2265–2270
- [4] “Study of nonlinear processes of a large experimental thermoacoustic-Stirling heat engine by using computational fluid dynamics”, G.Y. Yu, E.C. Luo, W. Dai, J.Y. Hu
J. Appl. Phys., 102 (7) (2007) (074901-074901–074901-074907)
- [5] “Basic treatment of onset conditions and transient effects in thermoacoustic Stirling engines”, A.T.A.M. de Waele *J. Sound. Vib.*, 325 (4–5) (2009), pp. 974–988
- [6] “Visualization observation of onset and damping behaviors in a traveling-wave thermoacoustic engine by infrared imaging” Bo Wang et al., *International Journal of Heat and Mass Transfer* 54, 5070 (2011)
- [7] <http://mysite.du.edu/~jcalvert/waves/acoucirc.htm>
- [8] www.score.uk.com
- [9] “Designing a Low-Cost, Electricity Generating Cooking Stove”, Riley, P.H., Saha, C., and Johnson, C.M., *Technology and Society Magazine IEEE*, summer 2010. Digital Object Identifier 10.1109/MTS.2010.937029, 1932–4529/10/\$26.00©2010IEEE.
- [10] <http://en.wiktionary.org/wiki/squegging>
- [11] “The technique of Radio Design” (Book chapter 4) Zepler, E. E, Chapman and Hall, London; Wiley and OSns, New York, (1943)
- [12] <http://books.google.co.uk/books?id=XiKgKdeBi6cC&pg=PA58&lpg=PA58&dq=squegging+radio+handbook&source=bl&ots=yBixfWbT87&sig=MflyaERVKB51Pv2EBzz2pdI4ajg&hl=en&sa=X&ei=Eni6T4yVFqPG0QXG8ZXoBw&ved=0CF8Q6AEwAg>
- [13] “Fishbone-like instability in a looped-tube thermoacoustic engine” *J. Acoust. Soc. Am.* Volume 128, Issue 4, pp. EL188-EL194 (2010); (7 pages) Zhibin Yu, Artur J. Jaworski, and Abdulrahman S. Abduljalil
- [14] “Thermoacoustics: A Unifying Perspective for Some Engines and Refrigerators”
Greg W. Swift *Acoustical Society of America through the American Institute of Physics*, 2002 - 300 pages
- [15] “Use of electrodynamic drivers in thermoacoustic refrigerators” *J. Acoust. Soc. Am.* Volume 107, Issue 2, pp. 827-832 (2000) Ray Scott Wakeland.
- [16] http://www.score.uk.com/research/Shared%20Documents/Score_Papers/tinaelectricaanalogues.zip